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Optimal Asset Allocation in Retirement: A Downside Risk Perspective

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ABSTRACT

Once an individual has retired, asset allocation becomes a critical investment decision. Unfortunately, there is no consensus on what the optimal allocation should be for retirees of varying age, gender, and risk tolerance. This study analyzes the allocation question through a focus on the downside risks created by uncertainty over investment returns and life expectancy. We find that the range of appropriate equity asset allocations in retirement is strikingly low compared with those of typical lifecycle and retirement funds now in the marketplace. In fact, for retirement portfolios whose primary goal is to minimize the risk of depletion and sustain withdrawals, optimal equity allocations range between 5% and 25%. This quite conservative level of equity holdings changes little even when we significantly change our assumptions on capital market returns. We even find that more aggressive equity allocations, those that still retain some focus on depletion risk but also seek to provide substantial bequests to heirs, are also relatively conservative. The study suggests, in short, that the higher equity allocations used in many popular retirement investment products today significantly underestimate the risks that these higher-volatility portfolios pose to the sustainability of retirees' savings and to the incomes they depend on.

1. Introduction

One of the tenets of financial planning for retirement is that an individual's exposure to higher-risk assets like stocks should decline as his or her retirement date nears. This less volatile, increasingly conservative asset allocation pattern makes intuitive sense because a major stock market decline around the time of a person's retirement could affect his or her ability to fund retirement or even to retire at all.

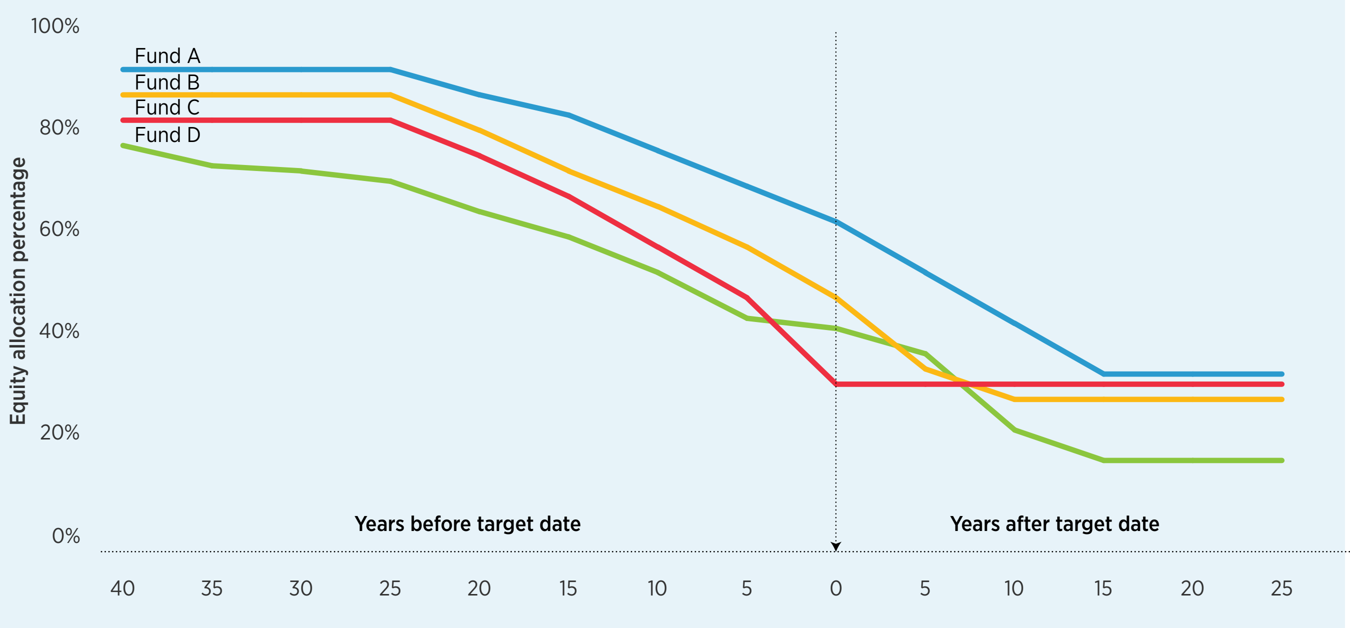
While this general concept is well accepted by investment professionals, there is no consensus as to what the exact asset allocation should be either at the moment of retirement or, for that matter, throughout retirement. (For purposes of clarity, we define "retirement" here as the moment when a person begins net draw-downs from their life savings to meet living expenses.)

This diversity of opinion among financial services providers shows itself clearly in the varied asset allocations offered by popular target-date or lifecycle funds. Exhibit 1 depicts the range of equity allocations over time for a few selected target-date funds. At the target date (presumably age 65), the allocations vary from a high of 65% to a low of 33%, with an average of 48%. Clearly the risk exposure for potential retirees would be significantly different depending on which of these investment products they choose for their retirement savings.

The purpose of this report is to take a closer look at this key decision for someone in retirement. To do so, we employ a unique set of analyses that summarizes the risk and return tradeoffs that go hand in hand with the asset allocation decision. For different levels of withdrawal amounts from retirement savings, we optimize the asset allocation mix in such a way so as to minimize the risk of plan failure, i.e., the depletion of funds. We then investigate how these allocations might change under varying sets of assets, assumptions, and retiree goals.

Our analyses suggest that when the focus is on avoiding retirement downside risk, the optimal asset allocation across a wide range of settings is strikingly conservative in terms of exposure to equities — far more conservative than those typically seen in the marketplace. Even in situations where individuals want to take more risk in order to increase the potential value of remaining assets to be left to their heirs, the range of allocations is still surprisingly conservative vis-à-vis conventional wisdom.

Exhibit 1. Selected target-date glide paths



Source: United States Government Accountability Office, Report GAO-11-118, "Defined Contribution Plans: Key Information on Target Date Funds as Default Investments Should Be Provided to Plan Sponsors and Participants," January 2011.

2. Methodology

Any individual faces multiple unknowns when planning for a retirement that could be 20 or 30 years or more. The most significant unknown variables are the future returns on retirement savings as well as the length of a person's life itself. When considering the investment decision, more aggressive asset allocations might have the potential to deliver higher average returns and thereby support longer retirement periods. Conversely, their higher risk and volatility also increase the danger of depleting assets early and causing the retirement plan to fail.

An attractive way to reduce this uncertainty and more accurately evaluate the financial tradeoffs and overall health of a retirement plan is through the use of a method known as Retirement Present Value, or "RPV," analysis. This approach assesses a retirement plan of current and future assets and liabilities. Savings contributions, for example, are both assets and flows into the portfolio. The value of these assets fluctuates with variable and uncertain investment returns over time. Retirement expenses, conversely, are both current and future liabilities reflected through outflows from the portfolio.

Of course, the duration of any specific plan or portfolio will vary because of the uncertainty of how long one will live. But RPV analysis captures and integrates all of these dynamic components — flows, returns, and longevity — and then discounts them into a positive or negative value expressed in today's dollars.

Rather than simulating returns to project the future value of a retirement portfolio (e.g., at age 85), the simulated returns are used as discount factors to compute the present value of future retirement cash flows. Mortality risk is captured by weighting these cash flows based on the probability of a person's being alive at any point in the future.[1] A positive RPV indicates the likelihood of having some assets left over at the end of life — the higher, the better. A negative RPV implies the possible or probable depletion of all retirement assets well before death — the lower the negative RPV, the worse.

For any given retirement plan, however, there is no single RPV value but rather a distribution of present values. This is because of the uncertainty of future investment returns compounded by the uncertainty of how long the individual will live. If the distribution of RPV results is completely positive (or nearly so), then we could expect a successful retirement outcome with a high degree of confidence. Conversely, a highly negative RPV distribution suggests a situation in which an individual is highly likely to outlive his or her retirement resources for the specific rate of withdrawal.

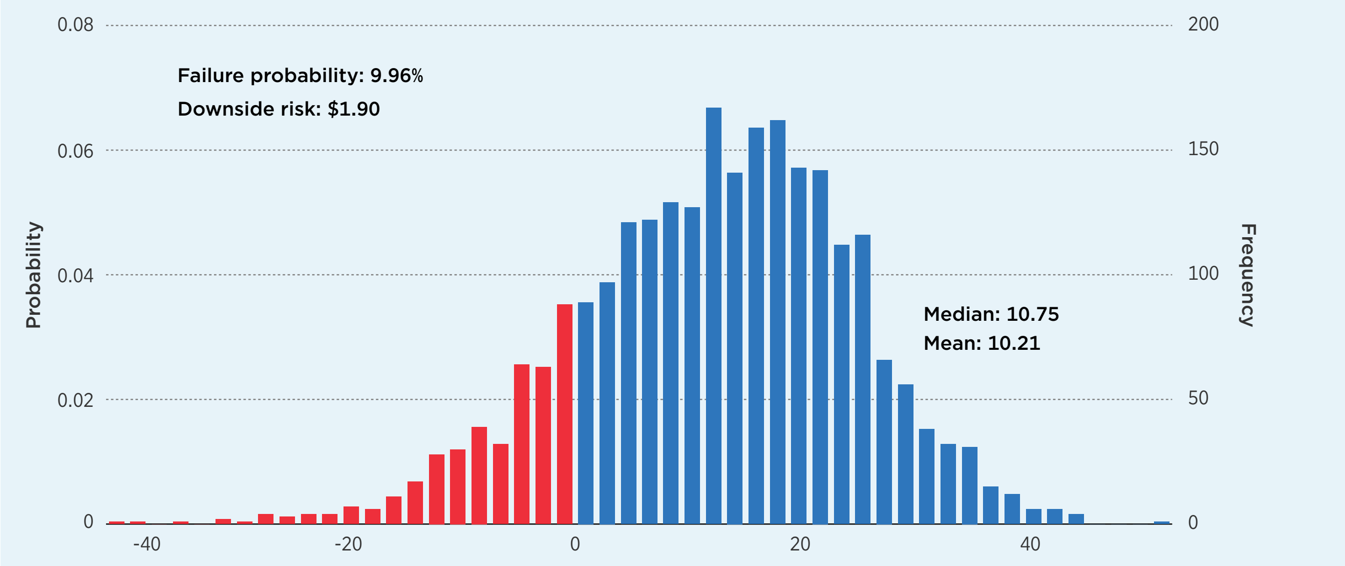
Leaving aside the issue of providing a bequest to heirs, the theoretically "perfect" retirement plan would be one in which the RPV would be exactly zero. In that unique case, a person would have precisely the right amount of retirement funds to spend before dying.

In reality, of course, individuals' retirement plans have a range of possible outcomes, from people outliving their resources to dying early and leaving a sizeable inheritance unspent. For planning purposes, one reasonable goal would be to reduce the possibility of a negative RPV (i.e., the probability of "ruin" or failure). An even more relevant goal might be to minimize the range or magnitude of portfolio "shortfalls." In other words, not only is the possibility of failure a concern, but we also want to minimize the severity of the risks represented by the possible negative RPVs that a portfolio is subject to.

To show how such an analysis works, Exhibit 2 provides the RPV distribution of an example retirement plan. In this case, a 65-year-old male has $100 in current retirement savings. He is retiring and plans to spend $7 per year in real terms. Throughout the retirement period, we assume that his retirement savings are invested in a constant mix of stock, bonds, and short-term instruments (we will refer to short-term instruments as simply "cash" from hereon). In this particular example, the allocation to stocks is 10%; bonds, 24%; and cash, 66% (this is, in fact, the allocation that minimizes retirement downside risk for this retiree). We also make the base-case assumptions that stock, bonds, and cash have real

1 In the analyses presented in this paper, retirement plan cash flows and simulated returns are estimated from the individual's current age out to age 110. Mortality effects are based on the Social Security Administration's period life tables.

Exhibit 2. RPV distribution for a retirement plan minimizes risk for a 65-year-old male with a $7 real spending rate per $100 in savings



Note: The analysis assumes that a 65-year-old individual has $100 in retirement savings and plans to spend $7 per year, adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled using the Social Security Administration's period life tables.

returns of 6.0%, 3.0%, and 1.0%, and volatilities of 16%, 7%, and 2.5%, respectively[2].

The RPV analysis for this example shows a wide distribution of possible outcomes. On average, the retirement plan has a value of $10.21 (median value of $10.75). Thus, in today's dollars, this is the net value of the plan — the present value of assets minus the present value of liabilities. An alternative interpretation of the average RPV is that it represents the amount that our retiree can expect to leave to heirs expressed in today's dollars. But as you can see, there is also a range of negative RPVs that represent unsuccessful retirement outcomes, i.e., total asset depletion. In fact, 9.96% of the outcomes have a negative present value. This represents a roughly one-in-ten chance of exhausting the portfolio's assets well before the retiree's death. Negative RPV outcomes can be thought of as situations in which the retiree has to borrow money from his heirs in order to support the desired spending level (or perhaps move in with them).

A more important statistic gleaned from this RPV analysis is the expected retirement downside risk of $1.90. This metric is based on the standard deviation of the negative RPVs weighted by the probability of them occurring — a measure called semi-deviation. This is a more valuable assessment of the severity of the downside risk than just the possibility of depletion because it captures the severity of the unsuccessful outcomes, some of which could be devastating.[3] For example, some outcomes shown in Exhibit 2 indicate adverse results as high as a negative $20, suggesting that there are combinations of market and mortality events that would have actually required 20% more in initial savings ($20 plus the original $100) at age 65 to completely fund a successful retirement at $7 per year.

These outlying cases could be the result of a combination of poor market returns early in the retirement plus an unexpectedly long retirement period due to extraordinary longevity. This "semi-deviation" measure of

2 We also make the base-case assumptions that real stock returns have a correlation with those of bonds and cash of 0.20 and 0.15, respectively, and that the correlation of real bond returns with cash returns is 0.35. These assumptions, as well as the expected returns and volatilities, are consistent with the historical evidence since 1946.

3 For a theoretical discussion of the relevance of downside risk measures in investment decision making, see "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence," (*Journal of Financial and Quantitative Analysis,* 1989) by W. V. Harlow and Ramesh K. S. Rao, and "Asset Allocation in a Downside Risk Framework," (*Financial Analysts Journal,* 1991) by W. V. Harlow.

downside risk helps to capture and measure how severe these negative outcomes can be.

Based on the distribution of RPV values illustrated in Exhibit 2, the three metrics just discussed provide a convenient way to summarize the financial characteristics and overall sustainability of a retirement plan. Retirement risk is captured by the probability of a shortfall (risk of ruin) and the standard deviation of shortfall. The overall health and net value of the plan is represented by the average RPV.

One final methodological issue needs to be discussed. The asset allocations used throughout this report are optimized so as to minimize retirement downside (depletion) risk for any given scenario. Given the complex nature of the problem we are examining, we are forced to use a stochastic optimization process to seek out the best asset allocation mix for any set of assumptions. This approach is different than that used for conventional optimization in that thousands of simulations are made with each step of the algorithm in its search for the best solution.

3. Minimum risk allocations

Let us first consider the allocation problem for an individual who is most concerned with achieving a successful retirement, i.e., minimizing the magnitude of any retirement failure. This goal is arguably the prime concern for most individuals. (It differs sharply from the goals of individuals who also wish to leave a bequest of assets to their heirs — cases we will also examine.)

Exhibit 3 provides the minimum risk allocations and retirement plan summary statistics for a wide range of scenarios. For both males and females, the table shows the optimal risk-minimizing asset allocations for retirees aged 65, 75, and 85. Three spending rates are shown for each gender and age. These withdrawal levels were chosen to reflect low, moderate, and high retirement expenses relative to a starting pool of $100 in retirement savings. The moderate spending rate was selected so that the probability of failure is around 10% (a level used by many retirement planning tools to reflect a reasonable and "sustainable" withdrawal amount). The low withdrawal case reflects a probability of failure less than 5%; the high withdrawal case, 20% to 30%. For example, as shown in the upper panel of the exhibit, a moderate spending rate for a 65-year-old male is $7 per $100 in savings. For 75- and 85-year-old males, the moderate spending rates are $11.50 and $22, respectively.

We can draw several important conclusions from Exhibit 3. Notice that all of the asset allocation mixes are quite conservative, with virtually all equity allocations less than 20%. For sustainable and low spending rates where the probability of failure is 10% or less, the equity allocations tend to be in the 5% to 10% range. These equity exposures are significantly lower than those we saw in Exhibit 1 for typical retirement products. In addition, notice that for the same level of risk, the spending rate for females is lower than that for males. For example, a 65-year-old male spending $7 has roughly the same risk and RPV profile as a 65-year-old female spending $6. The same is true at the $6 withdrawal amount for males and the $5 level for females. This is a simple reflection of the fact that females have a longer life expectancy and need their retirement savings to support a longer retirement. Alternatively, for the same spending rate, the equity allocation for females is higher to support the longer retirement. For example, at the $6 spending rate, the equity level is 11% for females versus 5% for males. For a $7 spending rate, the equity allocation is 21% for women versus 10% for men.

It is worth noting that these overall spending rates are higher than normally indicated for retirees by financial advisors. Often at age 65, a 4% or 5% spending rate is quoted as a rule of thumb that should sustain an individual's retirement. However, most financial planning tools do not incorporate the effects of mortality on expected spending levels. Here, with mortality included, a sustainable spending rate of $7 would be appropriate for males and $6 for females. On the other hand, if an individual expects to live to age 95, for example, the lower spending levels would be appropriate.[4]

One final observation from Exhibit 3 is the fact that the equity exposure does not change much throughout

4 Using our downside risk framework, a retiree, age 65 (male or female) who will live to age 95 with certainty, has a sustainable spending rate (10% probability of ruin) of $3.90 per $100 in savings and a risk-minimizing allocation to stocks, bonds, and cash of 12%, 31%, and 57%, respectively.

the retirement period. The equity allocations in the moderate spending case for a male are 10%, 11%, and 6% at ages 65, 75, and 85, respectively. For the female, they are 11%, 11%, and 6%. Thus, at least for the first part of retirement, they are fairly constant. On the other hand, the allocations to bonds and short-term interest instruments for both genders indicate a somewhat more conservative profile as age increases.

In thinking about asset allocation in retirement, most products and recommendations have only a modest amount of cash (short-term instruments). Certainly most employ far less than the optimal allocations in Exhibit 3. Often, cash is excluded from consideration or limited to 10% to 15%. Thus, the typical allocation decision is really one between stocks and bonds. As we see in Exhibit 3, however, a significant allocation to short-term interest

Exhibit 3. Asset allocations that minimize retirement downside risk — Three asset classes: stocks, bonds, and cash

Male						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$6	5%	20%	75%	0.20%	$0.11	$20.68
$7	10%	24%	66%	9.96%	$1.90	$10.22
$8	20%	46%	34%	32.96%	$7.15	$4.38
75 Year Old						
$11	9%	17%	74%	4.84%	$0.75	$9.81
$11.50	11%	21%	68%	14.00%	$1.82	$6.71
$12	14%	27%	59%	27.80%	$3.39	$4.13
85 Year Old						
$21	2%	6%	92%	0.60%	$0.13	$9.49
$22	6%	13%	81%	7.68%	$0.71	$6.38
$23	9%	19%	72%	24.60%	$2.07	$3.14

Female						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$5	2%	25%	73%	0.12%	$0.02	$23.34
$6	11%	24%	65%	7.80%	$1.78	$11.94
$7	21%	47%	32%	31.36%	$7.54	$5.29
75 Year Old						
$9	8%	17%	75%	2.28%	$0.46	$12.25
$9.50	11%	21%	68%	9.84%	$1.49	$8.58
$10	14%	27%	59%	23.00%	$3.18	$5.47
85 Year Old						
$17	2%	5%	93%	0.60%	$0.11	$10.31
$18	6%	14%	80%	8.08%	$0.82	$6.71
$19	10%	21%	69%	28.36%	$2.57	$2.95

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

instruments is needed to minimize the retirement downside risk across a large set of ages and spending rates. Let's now look at how the risk-minimizing allocations change when the role of such instruments is eliminated.

Exhibit 4 replicates the analyses from Exhibit 3 except with no allocation to cash. In this setting, there are some interesting observations to be made. First, notice that equity exposures rise to roughly 25% across all age, gender, and spending cases. Without cash to provide downside protection, the allocations to stocks increased since bonds do not provide as much protection against volatility.

Notice also that without cash in the mix, the overall level of retirement risk actually increases. With our base-case 65-year-old male and $7 spending rate, the risk metric from Exhibit 4 is $3.21 compared with $1.90 in the scenario when cash is in the solution — a 69% increase. However, when cash is excluded, the average RPV also increases throughout. This is an artifact of the higher expected returns of stocks and bonds relative to that of short-term instruments.

4. Sensitivity to investment assumptions

Clearly the ultimate success or failure of a retirement plan is closely tied to the returns and volatility of the assets in which we choose to invest retirement savings. While the minimum risk allocations that we saw in Exhibit 3 used very reasonable capital market assumptions based on long-term historical evidence, it is useful to test the findings with alternative sets of investment assumptions.

Returning now to our base case of a 65-year-old male spending $7, Exhibit 5 provides a comparison of the allocations, retirement risk, and RPV profile as assumptions are changed within the model. For example, in Scenario (1), the expected real return on stocks is increased from 6% to 7%. This results in an increase in the stock allocation to 14% versus the base-case allocation of 10%. Further, retirement risk decreases and the average RPV increases, reflecting the more attractive return expectations of stocks. Other scenarios look at the impact of changing the returns and volatilities of stocks and bonds as well as their correlations. Scenario (11) tests the combined effect of the three scenarios — (2), (5), and (8) — that result in a decrease in the equity allocation. Alternatively, Scenario (12) combines the five scenarios — (1), (4), (6), (7), and (10) — that increase the equity allocation.

The key conclusion that all of these scenarios suggest is that the conservative asset allocation result of the risk-minimizing portfolios is fairly robust to changes in the underlying assumptions. Even a combination of assumptions deliberately chosen to increase the aggressiveness of the allocation results in an equity level of just 22% — significantly below what is seen in typical retirement products.[5]

At this point, it is worth providing some intuition as to why the risk-minimizing portfolios have low equity allocations, in general, and remain low even in the various scenarios we have examined. The answer is linked to the primary cause of retirement shortfall, namely, sequence-of-returns risk. If a retiree is unfortunate enough to be exposed to a sequence of adverse returns early in retirement, the likelihood of an early depletion of savings rises dramatically. Such would have been the case for individuals retiring in 1973, 1999, or 2007, for example.

Any large exposure to equities carries with it an added chance of increasing this sequence-of-returns risk. While stocks' higher expected returns relative to bonds and cash are certainly an advantage for sustaining retirement savings, this benefit is outweighed by their potential for downside return shocks that increase the risk of ruin.

5 The set of scenarios in Exhibit 5 only reflect changes to the assumptions related to stocks and bonds. For the set of scenarios where stocks and bonds are assumed to be much more attractive than cash, the results have essentially already been presented in Exhibit 3.

Exhibit 4. Asset allocations that minimize retirement downside risk — Two asset classes: stocks and bonds only

Male						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$6	21%	79%	0%	1.16%	$0.94	$31.53
$7	23%	77%	0%	8.20%	$3.21	$20.54
$8	26%	74%	0%	24.36%	$7.73	$9.73
75 Year Old						
$11	25%	75%	0%	6.32%	$2.17	$18.33
$11.50	25%	75%	0%	11.84%	$3.27	$14.60
$12	25%	75%	0%	18.68%	$4.68	$10.99
85 Year Old						
$21	22%	78%	0%	4.08%	$1.42	$16.76
$22	22%	78%	0%	9.84%	$2.39	$12.76
$23	22%	78%	0%	18.20%	$3.76	$8.80

Female						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$5	22%	78%	0%	0.56%	$0.73	$35.38
$6	24%	76%	0%	6.44%	$3.03	$22.91
$7	27%	73%	0%	23.08%	$8.02	$10.68
75 Year Old						
$9	26%	74%	0%	4.36%	$1.77	$21.76
$9.50	25%	75%	0%	9.20%	$2.92	$17.31
$10	26%	74%	0%	15.76%	$4.44	$13.00
85 Year Old						
$17	23%	77%	0%	3.68%	$1.37	$18.61
$18	22%	78%	0%	10.00%	$2.53	$13.75
$19	22%	78%	0%	19.40%	$4.22	$9.01

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

Exhibit 5. Minimum risk allocations under different investment assumptions for a 65-year-old male with a $7 spending rate

Sensitivity Scenario	Scenario Description	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
	Base Case	10%	24%	66%	9.96%	$1.90	$10.22
1	Stock Return 6% → 7%	14%	24%	62%	7.16%	$1.62	$12.61
2	Stock Return 6% → 5%	7%	25%	68%	13.08%	$2.13	$8.45
3	Bond Return 3% → 3.5%	10%	32%	57%	6.84%	$1.58	$12.71
4	Bond Return 3% → 2.5%	11%	17%	72%	13.44%	$2.17	$8.49
5	Stock Volatility 16% → 18%	7%	25%	68%	11.76%	$2.06	$9.08
6	Stock Volatility 16% → 14%	15%	24%	61%	7.64%	$1.65	$12.00
7	Bond Volatility 7% → 8%	11%	16%	73%	12.44%	$2.11	$9.06
8	Bond Volatility 7% → 6%	10%	37%	53%	7.28%	$1.58	$12.04
9	Stock-Bond Correlation .20 → .30	10%	19%	70%	11.16%	$1.98	$9.26
10	Stock-Bond Correlation .20 → .10	12%	27%	61%	8.84%	$1.75	$11.48
11	Scenario (2)+(5)	5%	26%	69%	14.20%	$2.24	$7.89
12	Scenario (1)+(4)+(6)+(7)+(10)	22%	12%	66%	6.28%	$1.43	$14.04

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

5. Asset allocation with bequest objectives

In the final section of this report, we consider asset allocation for the subset of retired individuals who still have a concern for retirement risk but who also have a desire to leave assets to their heirs. As we discussed earlier, the average RPV of a retirement plan can be thought of as an estimate of the net value of a plan in today's dollars. Individuals who want to leave money to their heirs might be willing to take on some additional retirement risk in exchange for increasing the potential value of the assets remaining at the time of their death. In this context, there is, in fact, a continuous set of tradeoffs between retirement risk and RPV. So, just as there is an efficient frontier for investment securities that maximize the expected return for a given level of risk, there is an analogous efficient "retirement frontier" that best illustrates the tradeoffs between retirement risk and the value of potential bequest.

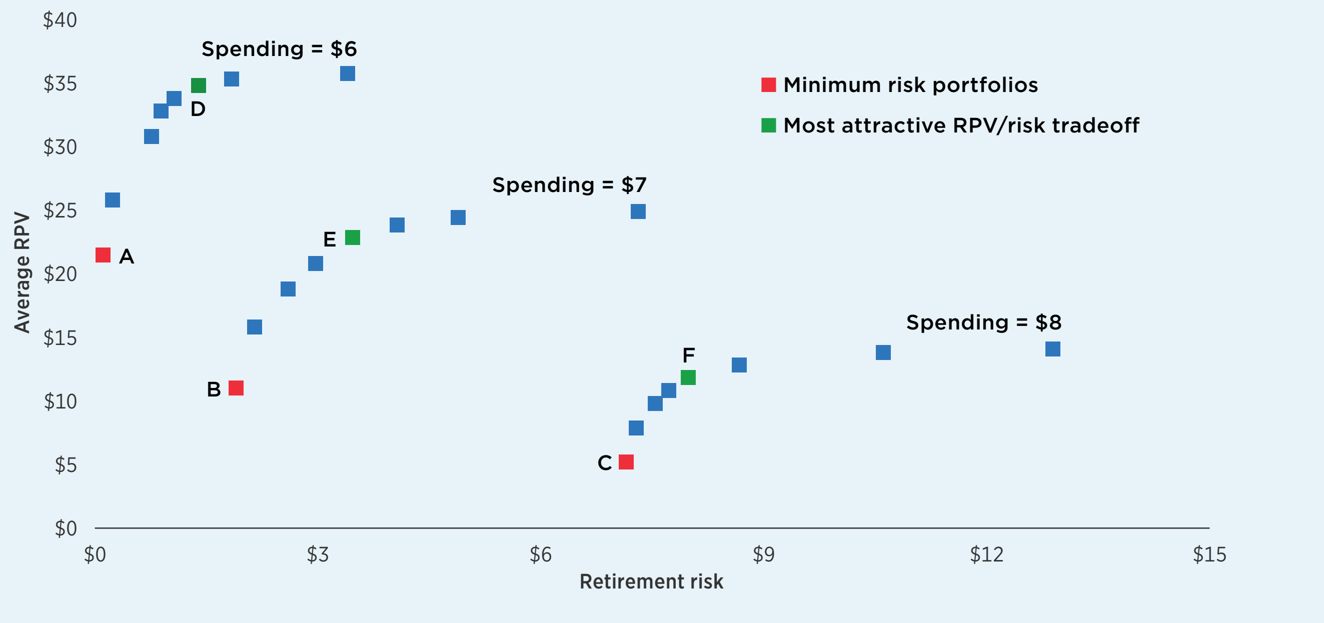
Exhibit 6 depicts the tradeoffs facing the individual who has both risk-control and bequest goals. The retirement frontiers are shown for a 65-year-old male with spending rates of $6, $7, and $8. The minimum risk portfolios are identified at the bottom left of each curve and are labeled as points A, B, and C. These portfolios and their characteristics were shown in Exhibit 3 and have equity allocations of 5%, 10%, and 20%, respectively. Once again, these portfolios are relevant for those individuals who are most concerned about the risk of outliving their retirement assets.

For each of the frontiers, as we move upward and to the right along the curves, retirement risk increases. However, with this added risk there is also an increase in the average RPV of the plan. Notice that initially, the curves are very steep — small increases in risk are accompanied by relatively large increases in the average RPV. In other words, in this region of the curve, the "cost" to increase the potential for a higher RPV is relatively low in terms of added retirement risk. Approximately midway through, the curves become almost flat. At this point, any increase in desired RPV results in very large increases in retirement risk. The marginal cost of increasing potential bequests, therefore, becomes very high.

In Exhibit 6, we have selected three portfolios, labeled D, E, and F, which would seem to reflect the upper limit of the RPV-risk tradeoff that would be attractive to most retirees. While this is a highly subjective selection, it does allow us to investigate the change in asset allocation that occurs as the retiree's objective moves beyond just a concern for the retiree's own risk to one that includes the desire to leave money to others. The allocations and portfolio characteristics for all points are provided in Appendix B.

As a starting point for this comparison, Exhibit 7 displays the RPV distribution for portfolio E (portfolio B for the same frontier is depicted in Exhibit 2). This portfolio has a stock, bond, and cash allocation of 34%, 66%, and 0%, respectively. While still a somewhat conservative balanced portfolio, its more aggressive positioning relative to the minimum risk portfolio, B, results in more than a doubling of the RPV from $10.21 to $22.02. Along with this increase in plan value, however, is an increase in retirement downside risk from $1.90 to $3.47 — an 83% increase. Interestingly, while downside risk increases, the probability of failure actually decreases slightly from 9.96% to 7.96%. Therefore, the likelihood of failure occurring decreases by 2%, but the semi-deviation metric indicates that when failure occurs, it is worse with the RPV values more highly negative.

Exhibit 6. RPV frontiers for a 65-year-old male

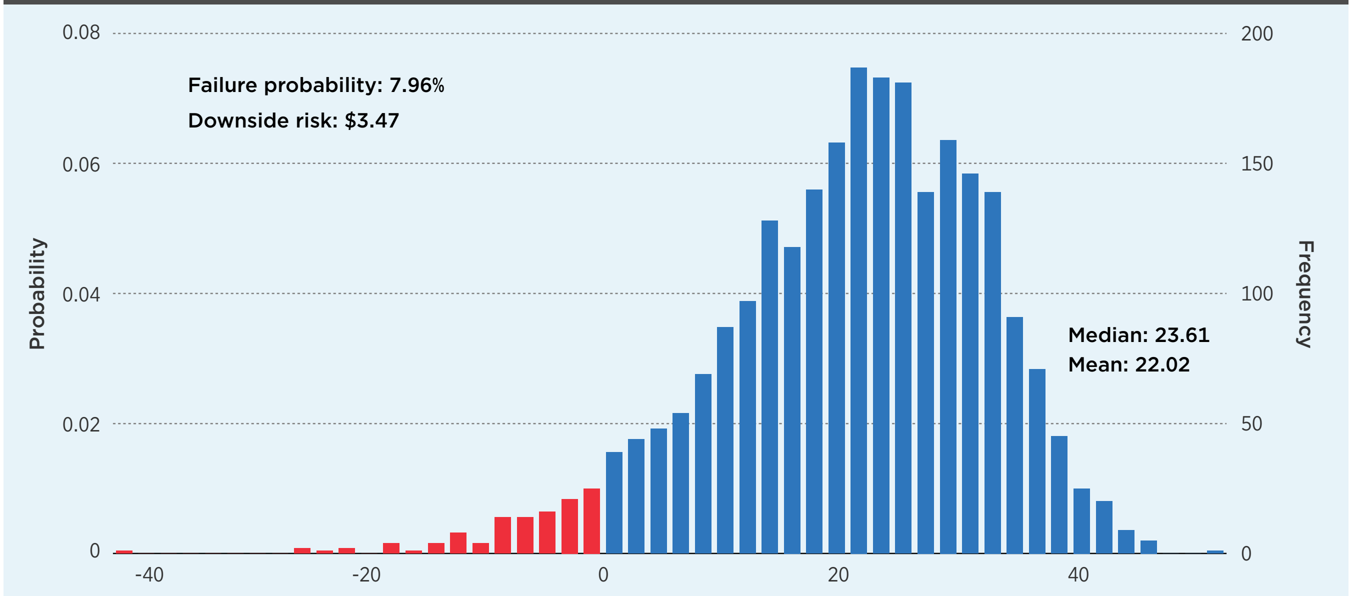


Note: The analysis assumes that a 65-year-old individual has $100 in retirement savings and plans to spend the indicated amount per year, adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled using the Social Security Administration's period life tables.

Exhibit 8 provides the asset allocation and RPV statistics for portfolios represented by points D, E, and F, as well as for similar portfolios chosen for different ages, gender, and spending rates. It is worth noting that the equity allocations for all of these portfolios are approximately twice that for those in the risk-averting examples we have seen earlier. Given these portfolios' bequest goal, their equity shares fall roughly in the 35% to 45% range.

While these results are computed based on using three asset classes — stocks, bonds, and cash — they also hold for the two-asset-class analysis since the allocation to cash is zero in all cases. The intriguing aspect of all of these results is that they are still more conservative than the typical allocations seen in financial products marketed to retirees even though we extended the risk positioning of the portfolios to increase the potential for bequest.

Exhibit 7. RPV distribution for an attractive asset allocation that considers both risk and bequest for a 65-year-old male with a $7 spending rate



Note: The analysis assumes that a 65-year-old individual has $100 in retirement savings and plans to spend $7 per year, adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled using the Social Security Administration's period life tables.

Exhibit 8. Asset allocations that consider both risk and bequest

Male						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$6	44%	56%	0%	1.80%	$1.40	$34.00
$7	34%	66%	0%	7.96%	$3.47	$22.02
$8	35%	65%	0%	22.92%	$7.98	$11.01
75 Year Old						
$11	42%	58%	0%	8.20%	$2.67	$20.01
$11.50	38%	62%	0%	11.64%	$3.62	$16.00
$12	34%	66%	0%	17.92%	$4.88	$12.00
85 Year Old						
$21	43%	57%	0%	6.20%	$1.89	$18.00
$22	41%	59%	0%	11.52%	$2.91	$14.01
$23	39%	61%	0%	18.96%	$4.27	$10.00

Female						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
65 Year Old						
$5	40%	60%	0%	0.72%	$0.99	$37.53
$6	39%	61%	0%	6.72%	$3.54	$25.00
$7	36%	64%	0%	22.24%	$8.29	$12.01
75 Year Old						
$9	38%	62%	0%	4.88%	$2.01	$23.01
$9.50	41%	59%	0%	9.84%	$3.40	$19.00
$10	34%	66%	0%	14.96%	$4.61	$14.00
85 Year Old						
$17	45%	55%	0%	5.48%	$1.93	$20.00
$18	39%	61%	0%	10.88%	$2.97	$15.01
$19	43%	57%	0%	19.88%	$5.01	$10.51

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

6. Conclusions

There are many ways to think about the risks of an individual's retirement plan, and how the asset allocation decision can influence those risks. The retirement present value, or RPV, provides a useful starting point by modeling the retirement plan as the net present value of assets minus liabilities weighted by the probability of survival. Because there is a distribution of RPVs based upon the realization of future investment returns and mortality events, risk can be thought of as the potential for negative outcomes in net plan value.

When minimizing the risk of retirement plan shortfalls, we find that the optimal asset allocation mix for sustainable spending rates is surprisingly conservative. Equity allocations for 65- to 85-year-old individuals are in the 5% to 10% range. With cash excluded from the asset mix, equity allocations for the minimum risk portfolios are still only around 25%. In addition, the allocations remain little changed even when we make substantial changes in the underlying investment risk and return assumptions. The conservative nature of the results differs significantly from most of the investment products for retirees in today's marketplace, which typically have an average equity allocation in excess of 45%.

Of course, not all retirees are focused only on minimizing the downside risk of their retirement plan. For some, taking on additional risk with a more aggressive asset allocation would be acceptable in exchange for the potential of leaving their heirs a larger estate. Even when we consider these tradeoffs, however, we still find that the optimal equity allocations are relatively conservative and in the 35% to 45% range.

Taken together, the results in this study should give any retiree pause before setting his or her asset allocation path in retirement. If mitigating the risk of outliving one's retirement resources is the cornerstone of the asset allocation decision, it is critical to limit equity exposure and recognize the impact that investment volatility can have on the sustainability of the retirement plan.

Appendix A

The retirement present value (RPV) is simply an expression of the financial value of a retirement plan in today's dollars. It captures both mortality risk and the uncertainty around investment returns by discounting the cash inflows and outflows of the retirement plan in the appropriate manner. If the discounted present value is positive, then it indicates a likelihood of having some assets left over at the end of life. A negative RPV implies the depletion of all of the assets well before death.

The calculation of the RPV is straightforward and merely an adaptation of the familiar method of determining the discounted present value of a series of future cash flows. Mathematically, the equation for the probability-weighted discounted cash flows is:

$$RPV = \sum_{t=0}^{\infty} \frac{p_t CF_t}{(1+R_t)^t}$$

where t = years into the future,

p_t = probability of being alive at time t,

CF_t = cash flow at time t, and

R_t = discount rate.

The cash flows of the retirement plan, CF_t, represent savings inflows into the portfolio prior to retirement age and the outflows from living expenses after retirement. CF_0 in the RPV analysis represents the individual's current savings at time $t = 0$.

For purposes of determining the discount rate, R_t, the returns on the investment portfolio in each year are used. These returns, denoted r_t, are obtained from historical time series or through Monte Carlo simulation. The discount rate is thus:

$$(1+R_t)^t = (1+r_1)(1+r_2)(1+r_3) \ldots (1+r_t)$$

The probability of being alive at time t, p_t, can be obtained directly from actuarial tables or through standard mathematical models specified to approximate the actual probability values.

Appendix B. RPV frontiers

Male — age 65						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
$6	75%	20%	5%	0.20%	$0.11	$20.68
$6	56%	30%	14%	0.28%	$0.24	$25.00
$6	10%	74%	16%	1.04%	$0.76	$30.00
$6	7%	64%	29%	0.92%	$0.89	$32.00
$6	2%	64%	34%	1.04%	$1.07	$33.00
$6	0%	56%	44%	1.80%	$1.40	$34.00
$6	0%	48%	52%	2.40%	$1.84	$34.51
$6	0%	31%	69%	4.40%	$3.40	$34.92
$7	66%	24%	10%	9.96%	$1.90	$10.21
$7	41%	41%	18%	8.20%	$2.15	$15.01
$7	20%	59%	21%	7.96%	$2.60	$18.00
$7	9%	66%	25%	7.76%	$2.97	$20.01
$7	0%	66%	34%	7.96%	$3.47	$22.02
$7	0%	56%	44%	8.60%	$4.07	$23.00
$7	0%	48%	52%	9.28%	$4.89	$23.60
$7	0%	31%	69%	12.20%	$7.31	$24.09
$8	34%	46%	20%	32.96%	$7.15	$4.38
$8	21%	55%	24%	28.24%	$7.28	$7.04
$8	9%	63%	28%	25.56%	$7.54	$9.01
$8	3%	66%	31%	24.40%	$7.72	$10.01
$8	0%	65%	35%	22.92%	$7.98	$11.01
$8	0%	56%	44%	22.80%	$8.67	$12.02
$8	0%	42%	58%	23.84%	$10.61	$13.00
$8	0%	30%	70%	25.20%	$12.89	$13.25

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

Appendix B. RPV frontiers

Female — age 65						
Spending Rate	Stocks	Bonds	Cash	Probability of Failure	Retirement Risk	RPV
$5	73%	25%	2%	0.12%	$0.02	$23.34
$5	48%	34%	18%	0.12%	$0.21	$30.01
$5	12%	62%	26%	0.44%	$0.59	$35.01
$5	5%	65%	30%	0.52%	$0.72	$36.08
$5	0%	65%	35%	0.64%	$0.88	$37.08
$5	0%	60%	40%	0.72%	$0.99	$37.53
$5	0%	54%	46%	1.24%	$1.18	$38.00
$5	0%	31%	69%	3.20%	$2.85	$38.82
$6	65%	24%	11%	7.80%	$1.78	$11.94
$6	51%	34%	15%	7.00%	$1.87	$15.04
$6	15%	66%	19%	6.68%	$2.57	$20.21
$6	7%	64%	29%	6.32%	$2.93	$23.02
$6	1%	67%	32%	6.36%	$3.16	$24.00
$6	0%	61%	39%	6.72%	$3.54	$25.00
$6	0%	48%	52%	7.92%	$4.57	$26.02
$6	0%	31%	69%	10.60%	$7.07	$26.56
$7	32%	47%	21%	31.36%	$7.54	$5.29
$7	20%	53%	27%	26.96%	$7.66	$8.01
$7	7%	65%	28%	24.36%	$7.88	$10.01
$7	2%	67%	31%	23.04%	$8.04	$11.01
$7	0%	64%	36%	22.24%	$8.29	$12.01
$7	0%	56%	44%	22.00%	$8.96	$13.03
$7	0%	42%	58%	22.80%	$10.83	$14.05
$7	0%	31%	69%	24.24%	$13.15	$14.32

Note: Spending rates represent the inflation-adjusted withdrawal rates per $100 in retirement savings. Retirement risk is measured by the semi-deviation of negative RPV outcomes.

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